THERMOELASTIC STRESSES IN A MULTILAYER PLATE UPON ACTION OF LASER RADIATION

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The problem of determining thermal stress in a multilayer plate upon action of a laser flux with Gaussian distribution is solved.

When a radiant flux acts upon a plate, a stress and deformation field is formed within the plate as a result of the heating produced. Of special interest is the case of stress and deformation fields in multilayer elements, the materials of which have markedly different values of thermal expansion coefficient and modulus of elasticity. In such a case even for low radiant flux intensity significant temperature stresses develop within the plate, which may reach the yield point.

In the present study we will assume that the intensity distribution over the cross section of the laser beam is Gaussian, and that energy absorption occurs on the irradiated surface. The problem of determining the temperature field in a multilayer plate consisting of n layers can be reduced to solution of the thermal conductivity equation with appropriate initial and final conditions

$$\frac{\partial T_i}{\partial t} = k_i\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_i}{\partial r}\right) + \frac{\partial^2 T_i}{\partial Z^2}\right], \tag{1}$$

$$T_i|_{t=0} = 0,\quad T_i|_{Z=H_i} = T_{i+1}|_{Z=H_i},\quad \lambda_i\frac{\partial T_i}{\partial Z}\Big|_{Z=H} = \lambda_{i+1}\frac{\partial T_{i+1}}{\partial Z}\Big|_{Z=H_i},$$
$$-\lambda_1\frac{\partial T_1}{\partial Z}\Big|_{Z=0} = I_0 e^{-ar^2},\quad \lambda_n\frac{\partial T_n}{\partial Z}\Big|_{Z=H} = 0. \tag{2}$$

where i is the layer number; $H_i = \sum_{k=0}^{i} d_k$, with d_k being the thickness of the layer with ordinal number k ($d_0 = 0$), and H being the plate thickness. Carrying out a Hankel transform of Eq. (1) and initial and final conditions (2), we obtain

$$\frac{\partial \overline{T}_i}{\partial t} = \frac{k_i}{H^2}\left(-\varepsilon^2 y^2 \overline{T}_i + \frac{\partial^2 \overline{T}_i}{\partial z^2}\right), \tag{3}$$

$$\overline{T}_i|_{t=0} = 0,\quad \overline{T}_i|_{z=h_i} = \overline{T}_{i+1}|_{z=h_i},\quad \lambda_i\frac{\partial \overline{T}_i}{\partial z}\Big|_{z=h_i} = \lambda_{i+1}\frac{\partial \overline{T}_{i+1}}{\partial z}\Big|_{z=h_i},$$
$$-\lambda_1\frac{\partial \overline{T}_1}{\partial z}\Big|_{z=0} = I_0\frac{H}{2a}e^{-y^2},\quad \lambda_n\frac{\partial \overline{T}_n}{\partial z}\Big|_{z=1} = 0,\quad h_i = \frac{H_i}{H}, \tag{4}$$

where p is a transform parameter; $z = Z/H$; $\varepsilon = 2\sqrt{a}H$; $y = p/2\sqrt{a}$, $\overline{T}_i(p, z, t) = \int_0^\infty T_i(r, z, t) r J_0(pr)\,dr$.

To transform to the original temperture we perform a reverse Hankel transform

$$T_i(r, z, t) = 4a\int_0^\infty \overline{T}_i(y, z, t) y J_0(2\sqrt{a}\,yr)\,dy. \tag{5}$$

It follows from boundary condition (4) at z = 0 that the thermal flux decays experimentally with increase in the parameter y. The basic contribution to the integral of Eq. (5) is produced by integration at low y values. Since the integrand contains a Chebyshev—Laguerre

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weight function, we can calculate the integral of Eq. (5) with the quadrature expression of [1] using a 12-point approximation, with the infinite upper limit replaced by $y \approx 3.3$.

To find the components of the stress tensor we make use of a system of differential equations for the displacements u, w in the radial and axial directions [2]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_i}{\partial r}\right)+\frac{\partial^2 u_i}{\partial z^2}-\frac{u_i}{r^2}+\frac{1}{1-2\nu_i}\frac{\partial e_i}{\partial r}=\frac{2(1+\nu_i)}{1-2\nu_i}\alpha_i\frac{\partial T_i}{\partial r}, \tag{6}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_i}{\partial r}\right)+\frac{\partial^2 w_i}{\partial z^2}+\frac{1}{1-2\nu_i}\frac{\partial e_i}{\partial z}=\frac{2(1+\nu_i)}{1-2\nu_i}\alpha_i\frac{\partial T_i}{\partial z}, \tag{7}$$

where $e_i = \partial u_i/\partial r + u_i/r + \partial w_i/\partial z$ is the volume expansion. In analogy to [3] we write u_i, w_i in the form

$$u_i=\int_0^\infty \varphi_i(z,\ p,\ t)\,pJ_1(pr)\,dp,$$
$$w_i=\int_0^\infty \Phi_i(z,\ p,\ t)\,pJ_0(pr)\,dp. \tag{8}$$

Substituting Eqs. (5), (8) in Eqs. (6), (7) we obtain a system of equations in the unknown functions φ_i, Φ_i:

$$\frac{d^2\varphi_i}{dz^2}-\frac{p}{1-2\nu_i}\frac{d\Phi_i}{dz}-\frac{2(1-\nu_i)}{1-2\nu_i}p^2\varphi_i=-\frac{2(1+\nu_i)}{1-2\nu_i}\alpha_i p\overline{T}_i, \tag{9}$$

$$\frac{2(1-\nu_i)}{1-2\nu_i}\frac{d^2\Phi_i}{dz^2}+\frac{p}{1-2\nu_i}\frac{d\varphi_i}{dz}-p^2\Phi_i=\frac{2(1+\nu_i)}{1-2\nu_i}\alpha_i\frac{\partial\overline{T}_i}{\partial z}. \tag{10}$$

Differentiating Eq. (9) with respect to z and considering Eq. (10):

$$\frac{d^2}{dz^2}\left(\frac{d\varphi_i}{dz}+p\Phi_i\right)-p^2\left(\frac{d\varphi_i}{dz}+p\Phi_i\right)=0.$$

We write the solution of this equation in the form

$$\frac{d\varphi_i}{dz}+p\Phi_i=A_i e^{-pz}+B_i e^{pz}, \tag{11}$$

where A_i, B_i are arbitrary constants. With consideration of Eq. (11), we obtain from Eq. (9),

$$\frac{d\Phi_i}{dz}+p\varphi_i=\alpha_i\frac{1+\nu_i}{1-\nu_i}\overline{T}_i+\frac{1-2\nu_i}{2(1-\nu_i)}(B_i e^{pz}-A_i e^{-pz}). \tag{12}$$

Adding and subtracting Eqs. (11), (12), we obtain a system of two ordinary differential equations, the solution of which we shall write in the form

$$\begin{gathered}\Phi_i=\frac{1}{2}\Big\{C_i e^{-p(z-h_{i-1})}+D_i e^{p(z-h_{i-1})}+\alpha_i\frac{1+\nu_i}{1-\nu_i}\Big(e^{-pz}\int_{h_{i-1}}^{z}\overline{T}_i e^{pz}dz+\\+e^{pz}\int_{h_{i-1}}^{z}\overline{T}_i e^{-pz}dz\Big)+\frac{A_i}{2(1-\nu_i)}\Big[(z-h_{i-1})e^{-pz}+\frac{3-4\nu_i}{2p}e^{pz}(e^{-2pz}-\\-e^{-2ph_{i-1}})\Big]+\frac{B_i}{2(1-\nu_i)}\Big[\frac{3-4\nu_i}{2p}e^{-pz}(e^{2pz}-e^{2ph_{i-1}})-(z-h_{i-1})e^{pz}\Big]\Big\},\end{gathered} \tag{13}$$

$$\begin{gathered}\varphi_i=\frac{1}{2}\Big\{C_i e^{-p(z-h_{i-1})}-D_i e^{p(z-h_{i-1})}+\alpha_i\frac{1+\nu_i}{1-\nu_i}\Big(e^{-pz}\times\\\times\int_{h_{i-1}}^{z}\overline{T}_i e^{pz}dz-e^{pz}\int_{h_{i-1}}^{z}\overline{T}_i e^{-pz}dz\Big)+\frac{A_i}{2(1-\nu_i)}\Big[(z-h_{i-1})e^{-pz}-\end{gathered}$$

$$-\frac{3-4\nu_i}{2p}e^{pz}\left(e^{-2pz}-e^{-2ph_{i-1}}\right)\Bigg]+\frac{B_i}{2(1-\nu_i)}\left[\frac{3-4\nu_i}{2p}e^{-pz}\times\right.$$
$$\left.\times\left(e^{2pz}-e^{2ph_{i-1}}\right)+(z-h_{i-1})e^{pz}\right]\Bigg\},\quad h_{i-1}\leqslant z\leqslant h_i. \tag{13}$$

We find the integration constants A_i, B_i, C_i, D_i from the boundary conditions on the free surfaces of the plate and the merger conditions for displacements and stresses at the interlayer boundaries

$$\sigma_{rz}=\sigma_{zz}=0,\ z=0;\ 1,\ u_i=u_{i+1},\ w_i=w_{i+1},\ (\sigma_{rz})_i=(\sigma_{rz})_{i+1},$$
$$(\sigma_{zz})_i=(\sigma_{zz})_{i+1},\ z=h_i. \tag{14}$$

We write the expressions for the components of the stress tensor $(\sigma_{rz})_i$, $(\sigma_{zz})_i$:

$$(\sigma_{rz})_i=G_i\left(\frac{\partial u_i}{\partial z}+\frac{\partial w_i}{\partial r}\right),\quad (\sigma_{zz})_i=2G_i\left[\frac{\partial w_i}{\partial z}+\frac{\nu_i}{1-2\nu_i}\left(\frac{\partial u_i}{\partial r}+\frac{u_i}{r}+\frac{\partial w_i}{\partial z}\right)-\frac{1+\nu_i}{1-2\nu_i}\alpha_i T_i\right]. \tag{15}$$

Substituting Eq. (8) in Eq. (15) with consideration of Eqs. (11), (12) we obtain

$$(\sigma_{rz})_i=G_i\int_0^\infty\left(\frac{d\varphi_i}{dz}-p\Phi_i\right)pJ_1(pr)\,dp=G_i\int_0^\infty(A_ie^{-pz}+B_ie^{pz}-2p\Phi_i)pJ_1(pr)dp,$$
$$(\sigma_{zz})_i=2G_i\int_0^\infty\left[\frac{1-\nu_i}{1-2\nu_i}\left(\frac{d\Phi_i}{dz}+p\varphi_i\right)-p\varphi_i-\alpha_i\frac{1+\nu_i}{1-2\nu_i}\overline{T}_i\right]\times \tag{16}$$
$$\times pJ_0(pr)\,dp=G_i\int_0^\infty(B_ie^{pz}-A_ie^{-pz}-2p\varphi_i)\,pJ_0(pr)\,dp.$$

From Eq. (14), by substituting Eqs. (13), (16) we obtain a system of 4n linear algebraic equations in the unknowns A_i, B_i, C_i, D_i:

$$B_1-\overline{C}_1=0,\quad A_1-\overline{D}_1=0,\quad B_i-g_iB_{i+1}-(1-g_i)\overline{C}_{i+1}\times$$
$$\times e^{-\varepsilon yh_i}=0,\quad A_i-g_iA_{i+1}-(1-g_i)\overline{D}_{i+1}e^{\varepsilon yh_i}=0,$$
$$A_i\frac{\varepsilon y(h_i-h_{i-1})}{2(1-\nu_i)}+B_i\frac{3-4\nu_i}{4(1-\nu_i)}\left(e^{2\varepsilon yh_i}-e^{2\varepsilon yh_{i-1}}\right)+$$
$$+\overline{C}_ie^{\varepsilon yh_{i-1}}-\overline{C}_{i+1}e^{\varepsilon yh_i}=-\alpha_i\frac{1+\nu_i}{1-\nu_i}\varepsilon y\int_{h_{i-1}}^{h_i}\overline{T}_ie^{\varepsilon yz}dz,$$
$$A_i\frac{3-4\nu_i}{4(1-\nu_i)}\left(e^{-2\varepsilon yh_i}-e^{-2\varepsilon yh_{i-1}}\right)-B_i\frac{\varepsilon y(h_i-h_{i-1})}{2(1-\nu_i)}+$$
$$+\overline{D}_ie^{-\varepsilon yh_{i-1}}-\overline{D}_{i+1}e^{-\varepsilon yh_i}=-\alpha_i\frac{1+\nu_i}{1-\nu_i}\varepsilon y\int_{h_{i-1}}^{h_i}\overline{T}_ie^{-\varepsilon yz}dz, \tag{17}$$
$$A_n\frac{\varepsilon y(1-h_{n-1})}{2(1-\nu_i)}+B_n\left[-e^{2\varepsilon y}+\frac{3-4\nu_n}{4(1-\nu_n)}\left(e^{2\varepsilon y}-e^{2\varepsilon yh_{n-1}}\right)\right]+\overline{C}_ne^{\varepsilon yh_{n-1}}=-\alpha_n\frac{1+\nu_n}{1-\nu_n}\varepsilon y\int_{h_{n-1}}^{1}\overline{T}_ne^{\varepsilon yz}dz,$$
$$A_n\left[e^{-2\varepsilon y}-\frac{3-4\nu_n}{4(1-\nu_n)}\left(e^{-2\varepsilon y}-e^{-2\varepsilon yh_{n-1}}\right)\right]+B_n\frac{\varepsilon y(1-h_{n-1})}{2(1-\nu_n)}-$$
$$-\overline{D}_ne^{-\varepsilon yh_{n-1}}=\alpha_n\frac{1+\nu_n}{1-\nu_n}\varepsilon y\int_{h_{n-1}}^{1}\overline{T}_ne^{-\varepsilon yz}dz,$$

where $g_i = G_{i+1}/G_i$, $\overline{C}_i = pC_i$, $\overline{D}_i = pD_i$.

By solving system (17) we obtain the values A_i, B_i, C_i, D_i as functions of the parameters ε, y. Substituting these coefficients in Eqs. (13), (16), we obtain the stress tensor components, while the radial and tangential components are defined by the following expressions:

$$(\sigma_{rr})_i = 2G_i\left[\frac{\partial u_i}{\partial r} + \frac{\nu_i}{1-2\nu_i}\left(\frac{\partial u_i}{\partial r} + \frac{u_i}{r} + \frac{\partial w_i}{\partial z}\right) - \frac{1+\nu_i}{1-2\nu_i}\alpha_i \times \right.$$
$$\left. \times T_i\right] = G_i\int_0^\infty y\left\{4\sqrt{a}\, y\varphi_i\left[J_0(y\bar{r}) - \frac{J_1(y\bar{r})}{y\bar{r}}\right] + \right.$$
$$\left. + \frac{\nu_i}{1-\nu_i}(B_i e^{\varepsilon yz} - A_i e^{-\varepsilon yz})J_0(y\bar{r})\right\}dy - 2G_i\frac{1+\nu_i}{1-\nu_i}\alpha_i T_i, \tag{18}$$
$$(\sigma_{\varphi\varphi})_i = 2G_i\left[\frac{u_i}{r} + \frac{\nu_i}{1-2\nu_i}\left(\frac{\partial u_i}{\partial r} + \frac{u_i}{r} + \frac{\partial w_i}{\partial z}\right) - \frac{1+\nu_i}{1-2\nu_i} \times \right.$$
$$\left. \times \alpha_i T_i\right] = G_i\int_0^\infty y\left\{4\sqrt{a}\,\varphi_i\frac{J_1(y\bar{r})}{\bar{r}} + \frac{\nu_i}{1-\nu_i}(B_i e^{\varepsilon yz} - \right.$$
$$\left. - A_i e^{-\varepsilon yz})J_0(y\bar{r})\right\}dy - 2G_i\frac{1+\nu_i}{1-\nu_i}\alpha_i T_i,$$

where $\bar{r} = 2\sqrt{a}r$. Thus, the problem of thermoelastic stress determination has been reduced to solution of the thermal conductivity equation in the one-dimensional approximation for a multilayer plate, Eqs. (3), (4), a system of 4n linear algebraic equations (17), and calculation of the integrals (16), (18).

Figure 1 shows the dependence of T, the plate surface temperature (z = 0), on radial coordinate $\bar{r}$ at various times. The solution was obtained for a two-layer Al–Ni plate (d_1 = 200 μm, d_2 = 800 μm) for parameter values $\varepsilon = 4$, $a = 4\cdot10^6$ m^{-2}. The quantity $W = \pi I_0/a$ is the radiant flux power absorbed on the plate surface.

Figure 2 shows the dependence of radial σ_{rr} and tangential $\sigma_{\varphi\varphi}$ components of the stress tensor on radial coordinate $\bar{r}$. On the metal contact surface these stresses are negative in Al, if $\bar{r} < 6$. This means that σ_{rr} and $\sigma_{\varphi\varphi}$ are compressive. With the passage of time they increase, even reaching significant values outside the region of laser beam focusing. In contrast to the contact surface, the rear surface of the plate is in tension near the beam axis $\bar{r} = 0$. With passage of time the stresses σ_{rr} change from positive to negative values. If the absorbed radiant power exceeds decades of W, the stresses will reach the yield point of the metals and specimen destruction will occur. Even in this case the temperature at any point within the plate is less than the melting point of Al or Ni.

Figure 3 shows the dependence of axial σ_{zz} and tangent σ_{rz} stress in the layer contact plane upon $\bar{r}$. The stress σ_{zz} near the axis $\bar{r} = 0$ is compressive and falls off rapidly with increase in $\bar{r}$, transforming to positive values. In contrast to σ_{rr} and $\sigma_{\varphi\varphi}$ outside the laser beam focusing region the axial stress practically vanishes. Calculations show that with decrease in the parameter a the axial stress σ_{zz} increases at an especially high rate. For example, at time t = 0.1 sec for $\varepsilon = 2$ ($a = 10^6$ m^{-2}) the stresses σ_{zz} (MPa) = −0.06 W (W), σ_{rr} = −3.85 W, while for $\varepsilon = 4$ ($a = 4\cdot10^6$ m^{-2}) σ_{zz} = −0.25 W, σ_{rr} = −5.5 W.

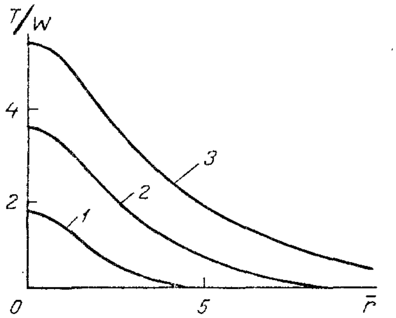
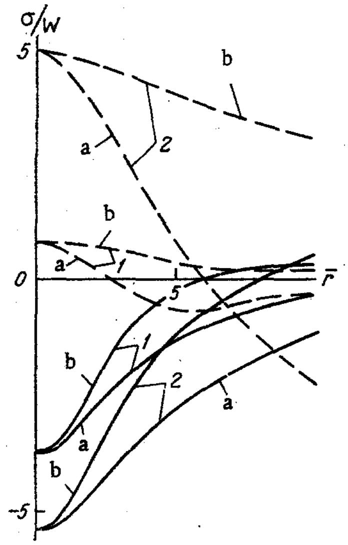


Fig. 1. Temperature T of plate face surface vs radial coordinate $\bar{r}$ for various times t: 1) t = 10^{-3}; 2) 10^{-2}; 3) 10^{-1} sec. T, K; W, W.



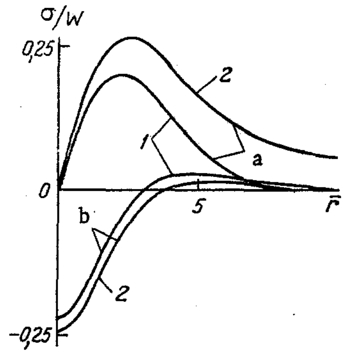


Fig. 2

Fig. 3

Fig. 2. Stresses σ_{rr} (a), $\sigma_{\varphi\varphi}$ (b) on layer contact surface (solid lines) and rear surface (dashes) of two-layer Al–Ni plate vs radial coordinate $\bar{r}$ for various times t: 1) t = 10^{-2}; 2) 10^{-1} sec. σ, MPa.

Fig. 3. Stresses σ_{rz} (a), σ_{zz} (b) on layer contact surface of two-layer Al–Ni plate vs radial coordinate $\bar{r}$ for various times t: 1) t = 10^{-2}; 2) t = 10^{-1} sec.

The results obtained can be used to determine limiting thermal flux values which cause destruction of a multilayer plate when exceeded.

NOTATION

T_i, layer temperature; λ_i, k_i, thermal conductivity and diffiusivity coefficients; r, Z; radial and axial coordinates; I_0, radiant flux density; $J_n(x)$, n-th-order Bessel function of the first sort; α_i, linear expansion coefficient; G_i, ν_i, shear modulus, Poisson coefficient.

LITERATURE CITED

1. V. I. Krylov and L. T. Shul'gina, Handbook of Numerical Integration [in Russian], Moscow (1979).
2. G. Parkus, Transient Thermal Stresses [in Russian], Moscow (1963).
3. G. I. Rudin, Inzh.-Fiz. Zh., 53, No. 1, 117-124 (1987).